On the Formation of Degenerate Heavy Neutrino Stars

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Abstract

The dynamics of a self-gravitating cold Fermi gas is described using the analogy with an interacting self-gravitating Bose condensate having the same Thomas-Fermi limit. The dissipationless formation of a heavy neutrino star through gravitational collapse and ejection of matter is demonstrated numerically. Such neutrino stars offer an alternative to black holes for the supermassive compact dark objects at the centers of galaxies.

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Supermassive neutrino stars, in which self-gravity is balanced by the degeneracy pressure of the cold fermions, have been a subject of speculation for more than three decades [1]. Originally, these objects were proposed as models for dark matter in galactic halos and clusters of galaxies, with neutrino masses in the $\sim$ eV range. More recently, however, degenerate superstars composed of weakly interacting fermions in the $\sim$ 10 keV range, have been suggested as an alternative to the supermassive black holes that are purported to exist at the centers of galaxies [2–6]. In fact, it has been shown [4] that such degenerate fermion stars could explain the whole range of supermassive compact dark objects which have been observed so far, with masses ranging from 10^6 to $3 \times 10^9 M_\odot$, merely assuming that a weakly interacting quasi-stable fermion of mass $m_f \simeq 15$ keV exists in nature.

As an example, the most massive compact dark object ever observed, is located at the center of M87, with a mass $M \simeq 3.2 \times 10^9 M_\odot$ [7]. Interpreting this object

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as a relativistic fermion star at the Oppenheimer-Volkoff [8] limit, yields a fermion mass of $m_f \simeq 15$ keV and a fermion star radius of $R = 4.45R_S \simeq 1.5$ light-days [3,4], where R_S is the Schwarzschild radius. In this case there is little difference between the fermion star and black hole scenarios, because the radius of the last stable orbit around a Schwarzschild black hole is $R = 3R_S$ anyway.

Extrapolating this down to the compact dark object at the center of our galaxy [9], which, having a mass $M \simeq 2.6 \times 10^6 M_\odot$, is at the lower limit of the mass range of the observed compact dark objects, we obtain, using the same fermion mass, a radius $R \simeq 20$ light-days $\simeq 5 \times 10^4 R_S$ [2]. As the potential inside such a nonrelativistic fermion star is shallow, the spectrum of radiation emitted by accreting baryonic matter is cut off for frequencies larger than 10^{13} Hz [3,5], as is actually observed in the spectrum of the enigmatic radio source Sgr A* at the galactic center [10]. A fermion star with radius $R \lesssim 20$ light-days and mass $M \simeq 2.6 \times 10^6 M_\odot$ is also consistent [6] with the observed motion of stars within a projected distance of 6 to 30 light-days from Sgr A* [9].

Of course, it is well-known that 15 keV lies squarely in the cosmologically forbidden mass range for stable active neutrinos ν [11]. The existence of such a massive active neutrino is also disfavoured by the Super-Kamiokande data [12]. However, as shown by Shi and Fuller [13] for an initial lepton asymmetry of $\sim 10^{-3}$, a sterile neutrino ν_s of mass $m_s \sim 10$ keV may be resonantly produced in the early universe with near closure density, i.e., $\Omega \simeq 1$. The resulting energy spectrum is not thermal but rather cut off so that it approximates a degenerate Fermi gas. Sterile neutrinos in this mass range are also constrained by astrophysical bounds on the radiative decay $\nu_s \rightarrow \nu\gamma$ [14]. However, the allowed parameter space includes $m_s \simeq 15$ keV, contributing $\Omega_s \simeq 0.3$ to the critical density, as favoured by the BOOMERANG data [15].

The statics of degenerate fermion stars is well understood, being the Oppenheimer-Volkoff equation in the relativistic case [8], or the Lané-Emden equation with polytropic index $n = 3/2$ in the nonrelativistic limit [16]. Alternatively, one may understand these as the Thomas-Fermi theory applied to self-gravitating fermion systems. The extension of the Thomas-Fermi theory to finite temperature [17,24] has been used to show that, at a certain critical temperature, weakly interacting massive fermionic matter undergoes a first-order gravitational phase transition from a diffuse to a clustered state, i.e. a nearly degenerate fermion star. However, such studies do not bear on the crucial dynamical question of whether the fermion star can form through gravitational collapse of density fluctuations in an orthodox cosmological setting. Indeed, since collisional damping is negligible, one would expect that only a virialized cloud results [11].

N-body simulations of the collisionless Boltzmann or Vlasov equation evidence

a rather different picture: the collapse is followed by a series of bounces, with matter expelled at each bounce, leaving behind a condensed object [19]. By Liouville's theorem, the Vlasov equation describes an incompressible fluid in phase-space so that it respects a form of the exclusion principle. Hence, these N-body simulations are effectively fermion simulations. What transpires is that gravity, being attractive, self-organises the phase-space fluid into a high-density/momentum core at the expense of other low-density/momentum regions as seen in the evolution of the spherical Vlasov equation [20].

Much the same behaviour is observed in the formation of mini-boson stars through so-called gravitational cooling [21]. Such a boson star is stable by balancing the uncertainty and gravitational pressures. A similar mechanism works in the presence of a quartic self-interaction [22], which dominates over uncertainty pressure, resulting in an equilibrium radius $R \gg 1/m$, where m is the boson mass [23]. Hence, we have a universal description of the physics underlying the formation process: once the collapse proceeds far enough, uncertainty, interaction or degeneracy pressure results in a bounce, in which the outgoing shock wave carries away the binding energy. The virial counter argument mentioned above is bypassed, because the ejected matter invalidates its assumption that there is no flow through the boundary.

The purpose of this letter is to verify this picture for the formation of a fermion star from a cold gravitationally unstable configuration. The dynamical Thomas-Fermi theory, developed long ago by Bloch for the electron gas [24], amounts to Euler's equations for irrotational flow, subject to an equation of state $P = P(\rho)$. The problem is that, due to the presence of shocks and instabilities, a naive integration of the Bloch equations is precluded in the gravitational case. The usual remedy is to introduce some small numerical viscosity. However, it seems imprudent to draw conclusions based on introducing an *ad-hoc* dissipation into what is fundamentally a dissipationless system. Here we take another, literally conservative approach.

We base our approach on the equivalence of a degenerate non-interacting fermion star with a certain type of self-interacting cold boson star. We shall demonstrate this equivalence using scaling arguments which are similar in spirit to the analysis of self-interacting boson stars by Colpi, Shapiro and Wassermann [23]. Consider a complex scalar field ψ with a repulsive Lagrangean interaction term $U(|\psi|^2)$. For simplicity, we use the Newtonian approximation, but our analysis may be easily extended in a general relativistic context. In the Newtonian limit, a self-interacting boson star is governed by the Gross-Pitaevskii-like equations

$$i\frac{\partial \psi}{\partial t} = \left[-\frac{\Delta}{2m} + \frac{1}{m}\frac{\mathrm{d}U}{\mathrm{d}|\psi|^2} + m\varphi \right] \psi, \tag{1}$$

$$\Delta\varphi = 4\pi G\rho, \tag{2}$$

$$\rho = m^2 |\psi|^2. \tag{3}$$

The pressure tensor is given by

$$P_{ij} = \mathrm{Re}\, \frac{\partial \psi}{\partial x_i} \frac{\partial \psi^*}{\partial x_j} + \delta_{ij} \left(|\psi|^2 \frac{\mathrm{d}U}{\mathrm{d}|\psi|^2} - U - \frac{\Delta |\psi|^2}{4} \right). \tag{4}$$

Following Colpi et al. [23], we introduce a dimensionless parameter Λ and define

$$R_* \equiv \frac{\Lambda}{m} \equiv \lambda \frac{m_{\mathrm{Pl}}}{m^2}; \quad M_* = R_* m_{\mathrm{Pl}}^2, \tag{5}$$

as the length and mass scales, respectively. Here $m_{\mathrm{Pl}} \equiv 1/\sqrt{G}$ denotes the Planck mass and the arbitrary constant λ will be fixed later using physical arguments. The substitution

$$\psi = \frac{m}{\sqrt{4\pi\lambda}} \Psi \tag{6}$$

yields the coupled dimensionless equations

$$\frac{i}{\Lambda} \frac{\partial \Psi}{\partial t} = \left[-\frac{\Delta}{2\Lambda^2} + \varphi + V(|\Psi|^2) \right] \Psi, \tag{7}$$

$$\Delta \varphi = |\Psi|^2, \tag{8}$$

where we have introduced the dimensionless potential

$$V(|\Psi|^2) = \frac{4\pi\lambda^2}{m^4} \frac{\mathrm{d}U}{\mathrm{d}|\Psi|^2}. \tag{9}$$

The pressure tensor (4), written in terms of dimensionless variables, reads

$$P_{ij} = \frac{m^4}{4\pi\lambda^2} \frac{1}{\Lambda^2} \left(\mathrm{Re}\, \frac{\partial \Psi}{\partial x_i} \frac{\partial \Psi^*}{\partial x_j} - \delta_{ij} \frac{\Delta |\Psi|^2}{4} \right) + \delta_{ij} \left(|\Psi|^2 \frac{\mathrm{d}U}{\mathrm{d}|\Psi|^2} - U \right). \tag{10}$$

A static, spherically symmetric solution, usually referred to as boson star, is obtained by the ansatz

$$\Psi = \mathrm{e}^{-i\epsilon R_* t} \Phi(r), \tag{11}$$

where $\Phi(r)$ is a real function. It is clear from the definition (5) that $\Lambda \gg 1$, as long as $m/m_{\mathrm{Pl}} \ll \lambda$ which is a reasonable assumption. Hence, for sufficiently large Λ, Eq. (7) with (11) degenerates to

$$\frac{\epsilon}{m} - \varphi - V(\Phi^2) = 0. \tag{12}$$

This equation is exact in the limit $\Lambda \to \infty$, which is just the Thomas-Fermi limit [25]. In this limit the derivative terms in Eq. (10) vanish so that the

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pressure tensor becomes diagonal, with all the components equal to $P \equiv P_{ii}$. We obtain an effective equation of state given by

$$\rho = \frac{m^4}{4\pi\lambda^2}\Phi^2, \tag{13}$$

$$P = \rho V - U. \tag{14}$$

It may be easily shown that the last equation combined with (12) yields the equation of hydrostatic equilibrium

$$\frac{dP}{\rho} = -d\varphi. \tag{15}$$

For a particular $U(|\psi|^2)$, the equation of state in the form $P = P(\rho)$ is obtained by eliminating Φ^2 from (13) and (14). On the other hand, if the equation of state, e.g., of the polytropic type, is given, we can determine the potential U and its derivative V by integrating (15). For a general polytropic equation of state

$$P(\rho) = K\rho^{1+1/n}, \tag{16}$$

Eq. (14) yields

$$V = (n+1)K\rho^{1/n} . \tag{17}$$

This, together with Eq. (13), gives the potential in terms of Φ^2. As we still have the freedom to choose the parameter λ conveniently, we fix λ so that the potential V in (7) takes a simple form

$$V = |\Psi|^{2/n}. \tag{18}$$

The polytropic equation of state with $n = 3/2$, together with the equation of hydrostatic equilibrium (15) and Poisson's equation (2), describes a degenerate fermion star. Hence, we have demonstrated that a degenerate fermion star is equivalent to a self-interacting boson star in the limit $\Lambda \to \infty$. Moreover, it may be shown numerically that, even for moderate values of Λ of the order of 10 to 20, the static solutions are almost degenerate and are quite well approximated by the static solution for an infinite Λ. This has been demonstrated for a quartic self-interaction [23] (note that our Λ corresponds to $\sqrt{\Lambda}$ in reference [23]).

Next we proceed to solving Eqs. (7), (8) and (18) numerically for large, but finite value of Λ. For weakly interacting degenerate fermions with the polytropic index $n = 3/2$, the length and mass scales are given by

$$R_* = \left(\frac{9\pi^2}{32g_f^2}\right)^{1/4}\frac{m_{\rm Pl}}{m_f^2} = 0.2325\left(\frac{\rm keV}{m_f}\right)^2\sqrt{\frac{2}{g_f}}\ {\rm lyr}\ , \tag{19}$$

$$M_* = 1.489 \left(\frac{\text{keV}}{m_f}\right)^2 \sqrt{\frac{2}{g_f}} \times 10^{12} \, M_\odot \,, \tag{20}$$

where g_f is the spin degeneracy and m_f the mass of the fermion. As M_* is of the order of the Oppenheimer-Volkoff limit, the validity of the Newtonian approximation in the static case requires

$$M = (4\pi)^{-1} \int \mathrm{d}^3 r \, |\Psi|^2 \ll 1. \tag{21}$$

By construction, once the overall scale is specified, a large but finite Λ allows us to simulate the fermionic problem as a bosonic one, as long as their Thomas-Fermi limits coincide, while providing an explicitly energy conserving way of controlling the shocks and instabilities. The basic regulating mechanism is the kinetic part of (7) which penalises density spikes. Of course, Λ must be so large that this term does not change the static scaling relationship [16]

$$MR^{\left(\frac{3-n}{n-1}\right)} = C_n, \tag{22}$$

arising from the polytropic equation of state. Our criterion is that the ratio of kinetic and pressure contributions to the static energy functional should be small. In particular for a Gaussian $\Psi = \alpha \exp\left[-(r/\beta)^2\right]$ we find

$$\frac{1}{2\Lambda^2} \frac{\int \mathrm{d}^3 r |\nabla \Psi|^2}{\int \mathrm{d}^3 r |\Psi|^{2+2/n}} = \left(1 + \frac{1}{n}\right)^{3/2} \frac{3}{2\Lambda^2 \beta^2 \alpha^{2/n}} \,. \tag{23}$$

For $n = 3/2$ this is independent of the size of β for a given mass, yielding the weak condition $M \gg 0.91\Lambda^{-3}$. When the mass violates this inequality the self-interaction V becomes irrelevant and there is a crossover to mini-boson star behaviour $MR = \text{const}$.

In Fig. 1 we display the evolution of $|r\Psi|^2$ for the spherical collapse of a mass $M = 0.008$, initially in the form of the above Gaussian with $\beta = 100 = \Lambda$. A fairly coarse sampling is used to make this contour plot, as this allows one to see the basic features of matter ejection without the complicated fine detail hiding it. Eqs. (7) and (8) are first written in terms of $\eta = r\Psi$, and then solved assuming spherical symmetry using the method of lines [26] for $r = 0$ to 1000. The region is divided into 4000 intervals, and a fourth order polynomial is fitted in each interval, with the function continuous up to its second derivatives across each interval. The time step is adjusted dynamically to achieve the desired accuracy of 10^{-6}. The boundary conditions at $r = 0$ and $r = 1000$ are reflective. To prevent matter from being artificially reflected by the boundary at $r = 1000$, we have introduced an r-dependent imaginary part to the potential, or 'sponge' [22], from $r = 700$ to 1000, which removes the ejected fermionic matter. The mass in the region remains constant to within the numerical integration error until $t = 5 \times 10^4$, which is a quarter of the simulation period, when the first ejected matter reaches the 'sponge'.

The expected features of bounce and ejection, leaving a condensed core, are evident. We obtain identical results using the Crank-Nicholson method.

A pertinent question is whether the Thomas-Fermi approximation is satisfied during the ejection process. Similar to WKB, the Thomas-Fermi limit is obtained when the scale of variation of density is small compared to that of the potential V, i.e.,

$$\xi^2 \equiv \frac{|\nabla \rho|^2}{\rho^2 \, 8 \, \mathrm{d}U/\mathrm{d}|\psi|^2} = \frac{|\nabla|\Psi||^2}{2\Lambda|\Psi|^{2+2/n}} \ll 1. \tag{24}$$

In Fig. 2 we exhibit $|r\Psi|^2$ and ξ versus r at time 1.22×10^4, where ejecta first develop. Evidently, the Thomas-Fermi condition, is well satisfied, except near sharp density minima. This is expected since in a pure Thomas-Fermi system evolving under Bloch's equations, the sharp density gradient would evolve into a shock, which violates the Thomas-Fermi condition.

In Fig. 3 we show $|r\Psi|^2$ on the time slice $t = 2 \times 10^5$ of our $\Lambda = 100$ simulation. The core mass $M = 0.0057$ and radius $R = 28$ are roughly commensurate with (22) and [16] $C_{3/2} = 132.3843$. Of course, at this point there is still evolution and oscillation of the core, similar to the mini-boson star case [21], with R varying between about 25 and 31. In general one can expect a persistent seismology with a period T of the order of the radius divided by the average speed of sound, $T \sim 51/M$, which agrees with the simulations. The seismology includes higher-order modes as well.

In summary, using a bosonic representation of the dynamical Thomas-Fermi theory for a self-gravitating gas, we have shown that nonrelativistic, degenerate and weakly interacting fermionic matter will form supermassive fermion stars through gravitational collapse accompanied by ejection. For a fermion mass of $m_f \simeq 15$ keV, and a final mass $M \simeq 2.6 \times 10^6 M_\odot$ such a superstar is consistent with the observations of the compact dark object at the center of our galaxy. A similar demonstration for the formation of such a star near the Oppenheimer-Volkoff limit, and the question of cosmology with degenerate dark matter, requires a general relativistic extension which is under development and will be reported elsewhere.

Acknowledgements

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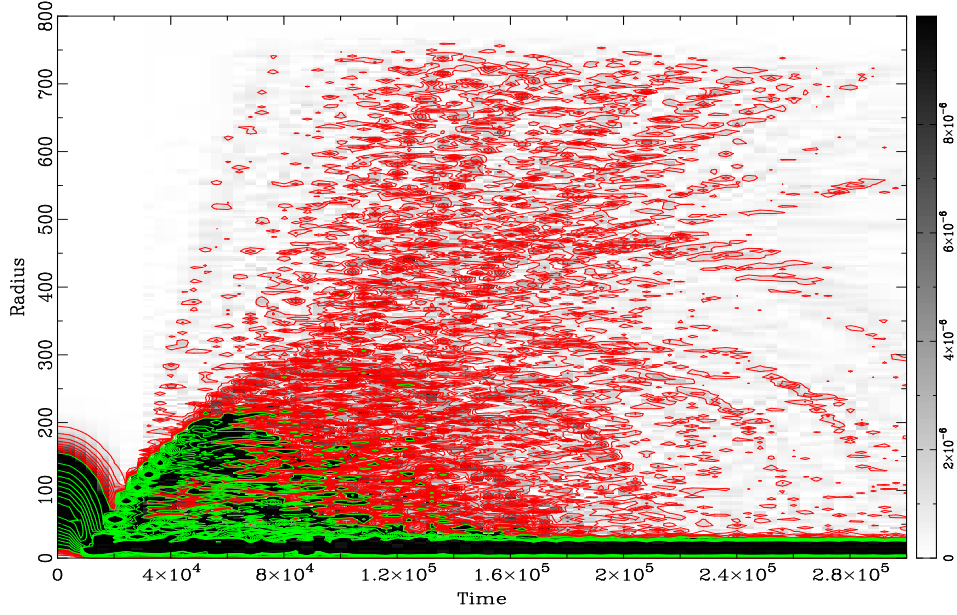
Fig. 1. Combined contour-density plot for the evolution of $|r\Psi|^2$ from the initial configuration described in the text. Green contour lines denote levels from 10^{-5} to 10^{-4} while red lines denote levels below 10^{-5}. Gravitational collapse is followed by ejection of excess matter leaving a fermion star at the center.
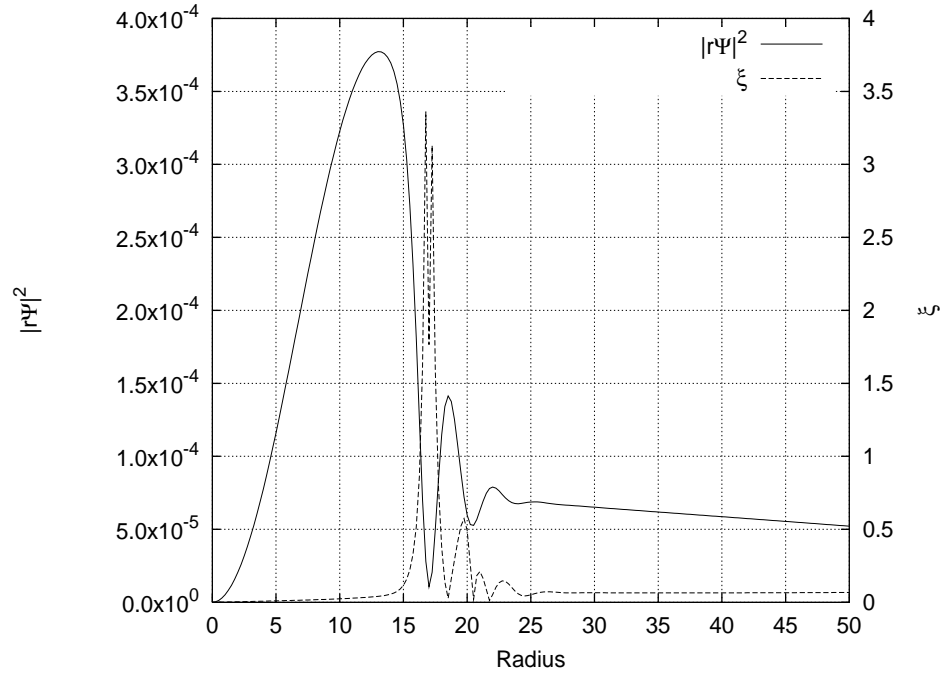

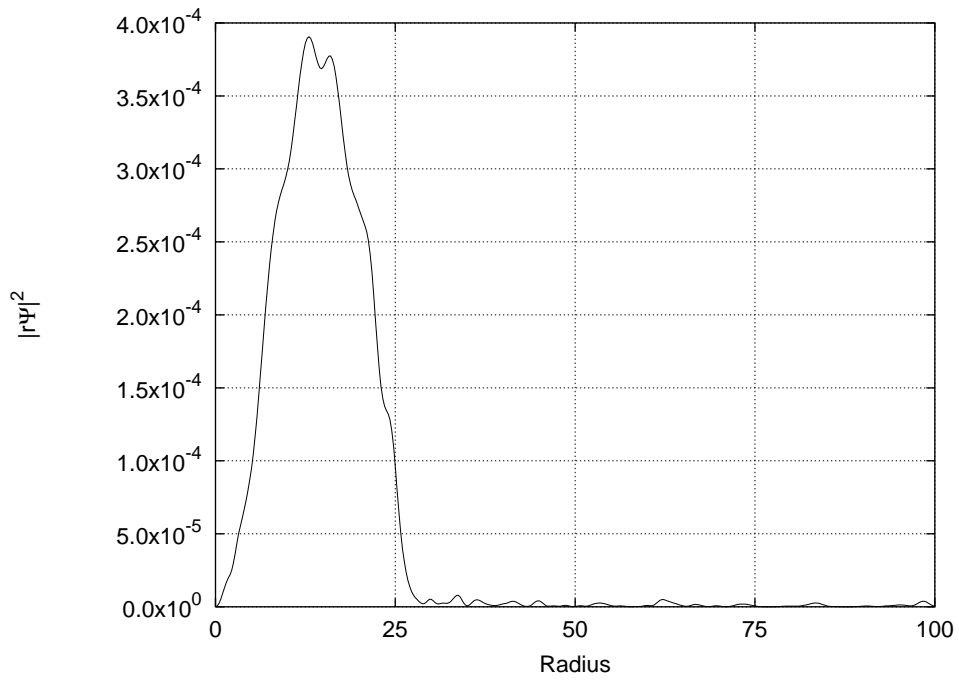Fig. 2. $|r\Psi|^2$ (solid line) and ξ (dashed line) versus r at $t = 1.22 \times 10^4$.

Fig. 3. $|r\Psi|^2$ versus r on the time slice $t = 2 \times 10^5$ in Figure 1.

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